Exercise 10

Differentiate.

$$J(v) = (v^3 - 2v)(v^{-4} + v^{-2})$$

Solution

Use the product rule to differentiate $J(v)$.

$$\begin{aligned}
J'(v) &= \frac{d}{dv}\left[(v^3 - 2v)(v^{-4} + v^{-2})\right] \\
&= \left[\frac{d}{dv}(v^3 - 2v)\right](v^{-4} + v^{-2}) + (v^3 - 2v)\left[\frac{d}{dv}(v^{-4} + v^{-2})\right] \\
&= (3v^2 - 2)(v^{-4} + v^{-2}) + (v^3 - 2v)(-4v^{-5} - 2v^{-3}) \\
&= (3v^{-2} + 3 - 2v^{-4} - 2v^{-2}) + (-4v^{-2} - 2 + 8v^{-4} + 4v^{-2}) \\
&= v^{-2} + 1 + 6v^{-4}
\end{aligned}$$

Alternatively, expand the function first

$$J(v) = v^{-1} + v - 2v^{-3} - 2v^{-1} = -v^{-1} + v - 2v^{-3}$$

and then differentiate it.

$$J'(v) = v^{-2} + 1 + 6v^{-4}$$